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MODLES OF HEAT CONDUCTION THROUGH RATE EQUATIONS

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ABSTRACT

In this talk we present a general constitutive scheme within continuum thermodynamics to describe the behavior of heat flow in deformable media. The rate-type constitutive equations are defined in the material (Lagrangian) description where the standard time derivative satisfies the principle of objectivity. All constitutive functions are required to depend on a common set of independent variables and to be consistent with thermodynamics. The statement of the Second Law is formulated in a general nonlocal form, where the entropy production rate is prescribed by a non-negative constitutive function and the extra entropy flux obeys a no-flow boundary condition. The thermodynamic response is then developed based on Coleman-Noll procedure. In the local formulation, the free energy potential and the rate of entropy production function are assumed to depend on temperature, temperature gradient and heat-flux vector along with their time derivatives. This approach results in rate-type constitutive equations for the heat-flux vector that are intrinsically consistent with the Second Law and easily amenable to analysis. A huge class of linear and nonlinear models of the rate type are recovered (e.g., Cattaneo-Maxwell's, Jeffreyslike, Green-Naghdi's, Quintanilla's and Burgers-like heat conductors). For each of these models some comments about the properties of the solutions of the corresponding temperature equation will be given. In particular, the stability of the solutions will be determined by the classical Routh-Hurwitz criterion. In the (weakly) nonlocal formulation of the second law, both the entropy production rate and an entropy extra-flux vector are assumed to depend on temperature, temperature gradient and heat-flux vector along with their spatial gradients and time derivatives. Within this classical thermodynamic framework the nonlocal Guyer-Krumhansl model and some nonlinear generalizations are obtained. This scheme allows the formulation of new models of heat transport that are likely to apply also in nanosystems.

References

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- [2] C. Giorgi, A. Morro, F. Zullo, Modeling of heat conduction through rate equations. Meccanica, 2024. *Meccanica*, Volume 59, pages 1757–1776, 2024